LESSON 8.4

Midsegments of Triangles

Common Core Math Standards

The student is expected to:

COMMON CORE **G-CO.C.10**

Prove theorems about triangles. Also G-CO.D.12, G-GPE.B.4, G-GPE.B.5

Mathematical Practices

COMMON CORE **MP.2 Reasoning**

Language Objective

Explain to a partner why a drawn segment in a triangle is or is not a midsegment.

ENGAGE

Essential Question: How are the segments that join the midpoints of a triangle's sides related to the triangle's sides?

Each midsegment is half the length of the side to which it is parallel; the sum of the lengths of the midsegments is half the perimeter of the triangle.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo, drawing attention to the horizontal crossbeams that extend between the slanting beams and stabilize them. Then preview the Lesson Performance Task.

Name ______________________ Class ____________ Date ________

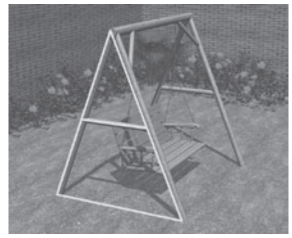
8.4 Midsegments of Triangles

Essential Question: How are the segments that join the midpoints of a triangle's sides related to the triangle's sides?

Resource Locker

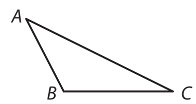
Explore Investigating Midsegments of a Triangle

The **midsegment** of a triangle is a line segment that connects the midpoints of two sides of the triangle. Every triangle has three midsegments. Midsegments are often used to add rigidity to structures. In the support for the garden swing shown, the crossbar $\overline{DE}$ is a midsegment of $\triangle ABC$

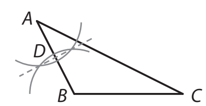


You can use a compass and straightedge to construct the midsegments of a triangle.

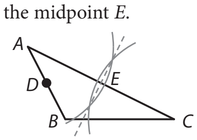
(A) Sketch a scalene triangle and label the vertices A, B, and C.



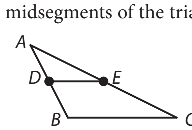
(B) Use a compass to find the midpoint of $\overline{AB}$. Label the midpoint D.



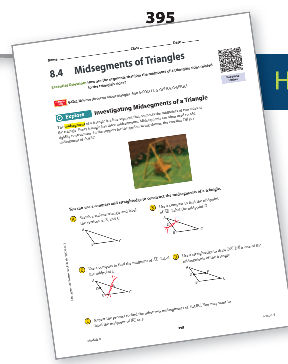
(C) Use a compass to find the midpoint of $\overline{AC}$. Label the midpoint E.



(D) Use a straightedge to draw $\overline{DE}$. $\overline{DE}$ is one of the midsegments of the triangle.



(E) Repeat the process to find the other two midsegments of $\triangle ABC$. You may want to label the midpoint of $\overline{BC}$ as F.



HARDCOVER PAGES 339–346

Turn to these pages to find this lesson in the hardcover student edition.

Reflect

1. Use a ruler to compare the length of $\overline{DE}$ to the length of $\overline{BC}$. What does this tell you about $\overline{DE}$ and $\overline{BC}$?
   The length of $\overline{DE}$ is half the length of $\overline{BC}$.

2. Use a protractor to compare $m\angle ADE$ and $m\angle ABC$. What does this tell you about $\overline{DE}$ and $\overline{BC}$? Explain.
   $m\angle ADE = m\angle ABC$, so $\overline{DE} \parallel \overline{BC}$ since corresponding angles are congruent.

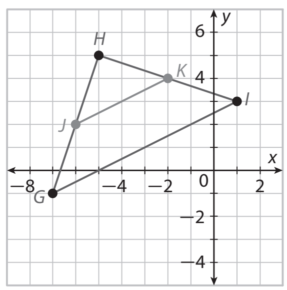
3. Compare your results with your class. Then state a conjecture about a midsegment of a triangle.
   A midsegment of a triangle is parallel to the third side of the triangle and is half as long as the third side.

Explain 1 Describing Midsegments on a Coordinate Grid

You can confirm your conjecture about midsegments using the formulas for the midpoint, slope, and distance.

Example 1 **Show that the given midsegment of the triangle is parallel to the third side of the triangle and is half as long as the third side.**

Ⓐ The vertices of $\triangle GHI$ are $G(-7, -1)$, $H(-5, 5)$, and $I(1, 3)$. J is the midpoint of $\overline{GH}$, and K is the midpoint of $\overline{IH}$. Show that $\overline{JK} \parallel \overline{GI}$ and $JK = \frac{1}{2}GI$. Sketch $\overline{JK}$.



Step 1 Use the midpoint formula, $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, to find the coordinates of J and K.

The midpoint of $\overline{GH}$ is $\left(\frac{-7-5}{2}, \frac{-1+5}{2}\right) = (-6, 2)$. Graph and label this point J.

The midpoint of $\overline{IH}$ is $\left(\frac{-5+1}{2}, \frac{5+3}{2}\right) = (-2, 4)$. Graph and label this point K. Use a straightedge to draw $\overline{JK}$.

Step 2 Use $\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$ to compare the slopes of $\overline{JK}$ and $\overline{GI}$.

Slope of $\overline{JK} = \frac{4-2}{-2-(-6)} = \frac{1}{2}$ Slope of $\overline{GI} = \frac{3-(-1)}{1-(-7)} = \frac{1}{2}$

Since the slopes are the same, $\overline{JK} \parallel \overline{GI}$.

Step 3 Use $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to compare the lengths of $\overline{JK}$ and $\overline{GI}$.

$JK = \sqrt{(-2-(-6))^2 + (4-2)^2} = \sqrt{20} = 2\sqrt{5}$

$GI = \sqrt{(1-(-7))^2 + (3-(-1))^2} = \sqrt{80} = 4\sqrt{5}$

Since $2\sqrt{5} = \frac{1}{2}(4\sqrt{5})$, $JK = \frac{1}{2}GI$.

EXPLORE

Investigating Midsegments of a Triangle

INTEGRATE TECHNOLOGY

Students have the option of completing the midsegments activity either in the book or online.

QUESTIONING STRATEGIES

How are the length of the midsegment and the third side of the triangle related? **The midsegment is one-half the length of the third side.**

EXPLAIN 1

Describing Midsegments on a Coordinate Grid

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Remind students that two lines with equal slopes are parallel and that the slope of a segment is the difference of its y-coordinates divided by the difference of its x-coordinates. Also remind them to subtract the coordinates in the same order.

PROFESSIONAL DEVELOPMENT

Math Background

A *midsegment* of a triangle is a segment that joins the midpoints of two sides of the triangle. Together, the three midsegments of a triangle form the sides of the *midsegment triangle*. Using the Triangle Midsegment Theorem and the SSS Triangle Congruence Theorem, it can be proven that the four small triangles formed by the midsegments are congruent. Since the four triangles together form the original triangle, each small triangle has one-fourth of its area.

QUESTIONING STRATEGIES

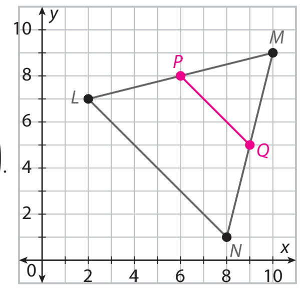
To which side of the triangle does the midsegment appear to be parallel? **the side that does not contain the endpoints of the midsegment**

How do you find the midsegments of a triangle in the coordinate plane? **Use the Midpoint Formula to find the coordinates of the midpoint of two sides of the triangle. Plot the midpoints and connect them to form a midsegment.**

What are some ways you can show that two line segments are parallel when using the Triangle Midsegment Theorem in a coordinate plane? **Sample answer: Show that the lines have the same slope.**

Ⓑ The vertices of $\triangle LMN$ are $L(2, 7)$, $M(10, 9)$, and $N(8, 1)$. P is the midpoint of $\overline{LM}$, and Q is the midpoint of $\overline{MN}$.

Show that $\overline{PQ} \parallel \overline{LN}$ and $PQ = \frac{1}{2}LN$. Sketch $\overline{PQ}$.



Step 1 The midpoint of $\overline{LM} = \left(\frac{2 + \boxed{10}}{2}, \frac{7 + \boxed{9}}{2}\right) = \left(\boxed{6}, \boxed{8}\right)$.
Graph and label this point P.

The midpoint of $\overline{NM} = \left(\frac{\boxed{8} + \boxed{10}}{2}, \frac{\boxed{1} + \boxed{9}}{2}\right)$

$= \left(\boxed{9}, \boxed{5}\right)$. Graph and label this point Q. Use a straightedge to draw $\overline{PQ}$.

Step 2 Slope of $\overline{PQ} = \frac{5 - 8}{9 - \boxed{6}} = \boxed{-1}$ Slope of $\overline{LN} = \frac{\boxed{1} - \boxed{7}}{\boxed{8} - \boxed{2}} = \boxed{-1}$

Since the slopes are the same, $\overline{PQ}$ and $\overline{LN}$ are **parallel**.

Step 3 $PQ = \sqrt{\left(\boxed{9} - 6\right)^2 + (5 - 8)^2} = \sqrt{\boxed{9} + 9} = \sqrt{18} = 3\sqrt{2}$

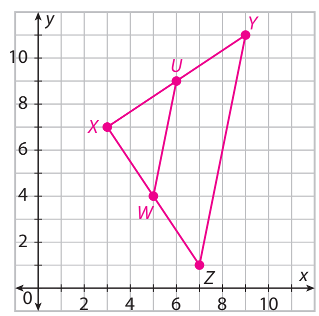
$LN = \sqrt{\left(\boxed{8} - \boxed{2}\right)^2 + \left(\boxed{1} - \boxed{7}\right)^2} = \sqrt{\boxed{36} + \boxed{36}} = \sqrt{\boxed{72}} = 6\sqrt{\boxed{2}}$

Since $\boxed{3}\sqrt{\boxed{2}} = \frac{1}{2}\left(\boxed{6}\sqrt{\boxed{2}}\right)$, $\boxed{PQ} = \frac{1}{2}\boxed{LN}$.

The length of $\overline{PQ}$ is **half** the length of $\overline{LN}$.

Your Turn

4. The vertices of $\triangle XYZ$ are $X(3, 7)$, $Y(9, 11)$, and $Z(7, 1)$. U is the midpoint of $\overline{XY}$, and W is the midpoint of $\overline{XZ}$. Show that $\overline{UW} \parallel \overline{YZ}$ and $UW = \frac{1}{2}YZ$. Sketch $\triangle XYZ$ and $\overline{UW}$.



$U = \left(\frac{3 + 9}{2}, \frac{7 + 11}{2}\right) = (6, 9)$;

$W = \left(\frac{3 + 7}{2}, \frac{7 + 1}{2}\right)$

$= (5, 4)$;

slope of $\overline{UW} = \frac{4 - 9}{5 - 6} = 5$; slope of $\overline{YZ} = \frac{1 - 11}{7 - 9} = 5$;

Since the slopes are the same, $\overline{UW} \parallel \overline{YZ}$.

$UW = \sqrt{(5 - 6)^2 + (4 - 9)^2} = \sqrt{26}$;

$YZ = \sqrt{(7 - 9)^2 + (1 - 11)^2} = 2\sqrt{26}$;

so $UW = \frac{1}{2}YZ$.

COLLABORATIVE LEARNING

Small Group Activity

Have one student in each group use geometry software to construct the midsegment of a triangle. Ask another student in the group to measure the midsegment and the sides of the triangle and make a conjecture about the relationship of the two measures. Ask a third student to measure the angles to make a conjecture about midsegment and the third side of the triangle. Have the fourth student drag the vertices of the triangle to verify the conjectures made by the other students: the midsegments are half the measure of the third side and are parallel to the third side.

Explain 2 Using the Triangle Midsegment Theorem

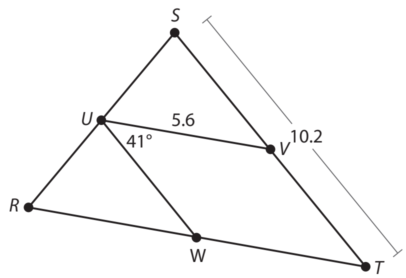
The relationship you have been exploring is true for the three midsegments of every triangle.

Triangle Midsegment Theorem

The segment joining the midpoints of two sides of a triangle is parallel to the third side, and its length is half the length of that side.

You explored this theorem in Example 1 and will be proving it later in this course.

Example 2 Use triangle *RST*.



(A) Find *UW*.

By the Triangle Midsegment Theorem, the length of midsegment $\overline{UW}$ is half the length of $\overline{ST}$.

$UW = \frac{1}{2}ST$

$UW = \frac{1}{2}(10.2)$

$UW = 5.1$

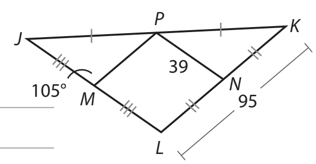
(B) Complete the reasoning to find $m\angle SVU$.

△Midsegment Thm.	$\overline{UW} \parallel \overline{ST}$
Alt. Int. Angles Thm.	$m\angle SVU = m\angle VUW$
Substitute **41°** for **m∠VUW**	$m\angle SVU =$ **41°**

Reflect

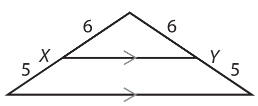
5. How do you know to which side of a triangle a midsegment is parallel?
Since a midsegment connects the midpoints of two sides, it is parallel to the third side.

Your Turn



6. Find *JL*, *PM*, and $m\angle MLK$.
$JL = 2(PN) = 2(39) = 78$; $PM = \frac{1}{2}KL = \frac{1}{2}(95) = 47.5$;
$m\angle MLK = m\angle JMP = 105°$

Elaborate



7. **Discussion** Explain why $\overline{XY}$ is NOT a midsegment of the triangle.
The endpoints of $\overline{XY}$ are not midpoints of the sides of the triangle.

EXPLAIN 2

Using the Triangle Midsegment Theorem

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Point out that if the midsegments of a triangle are found, then the Triangle Midsegment Theorem can be used to state that the midsegment is half the length of the third side and is parallel to the third side. Explain that the theorem gives a way to indirectly find the side lengths of a triangle, and that it gives similar triangles, which will be studied in Unit 4.

QUESTIONING STRATEGIES

? In the example, how do you know that you can use the Triangle Midsegment Theorem? **The midsegment of a triangle was found, so the Triangle Midsegment Theorem applies.**

AVOID COMMON ERRORS

Sometimes students write the incorrect algebraic expressions to indirectly find the lengths of segments in triangles when using the Triangle Midsegment Theorem. Have these students use the theorem to write the relationships among the segments before they substitute algebraic expressions for the segments. The correct expressions should give the correct equations to solve for the lengths.

DIFFERENTIATE INSTRUCTION

Modeling

Have students cut a large scalene triangle from heavy paper. Have them construct the midpoints of two sides and connect them, forming a *midsegment*. Then have them cut along this segment to form a triangle and a trapezoid. Have them rotate the small triangle and place it next to the trapezoid to form a parallelogram. Ask students to make conjectures about the relationship between the midsegment and the base of the triangle.

ELABORATE

QUESTIONING STRATEGIES

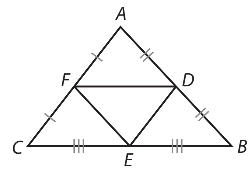
How could you use a property of midsegments to find a measurement for something that is difficult to measure directly? **Sample answer: If you cannot measure directly across something like a pond, you can construct a triangle around it so that the distance you want to measure is a midsegment of the triangle. Then you can measure the third side of the triangle and divide it by 2 to find the measure you are looking for.**

SUMMARIZE THE LESSON

Have students fill out a chart to summarize what they know about triangle midsegments. **Sample:**

Triangle Midsegment	
Definition	**A line segment that connects the midpoints of two sides of the triangle**
Properties	**Half the length of the third side; parallel to the third side**
Example	**Answers will vary.**
Non-example	**Answers will vary.**

8. **Essential Question Check–In** Explain how the perimeter of $\triangle DEF$ compares to that of $\triangle ABC$.



The perimeter of $\triangle DEF = \frac{1}{2}$ (perimeter of $\triangle ABC$).

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. Use a compass and a ruler or geometry software to construct an obtuse triangle. Label the vertices. Choose two sides and construct the midpoint of each side; then label and draw the midsegment. Describe the relationship between the length of the midsegment and the length of the third side.

   Drawings will vary. Students should conclude that the midsegment is half the length of the third side.

2. The vertices of $\triangle WXY$ are $W(-4, 1)$, $X(0, -5)$, and $Y(4, -1)$. A is the midpoint of $\overline{WY}$, and B is the midpoint of $\overline{XY}$. Show that $\overline{AB} \parallel \overline{WX}$ and $AB = \frac{1}{2}WX$.

   The coordinates of A and B are $(0, 0)$ and $(2, -3)$. The slope of $\overline{AB}$ and $\overline{WX}$ is $-\frac{3}{2}$, so the lines are parallel. The length of $\overline{AB}$ is $\sqrt{13}$ and the length of $\overline{WX}$ is $2\sqrt{13}$, so $AB = \frac{1}{2}WX$.

3. The vertices of $\triangle FGH$ are $F(-1, 1)$, $G(-5, 4)$, and $H(-5, -2)$. X is the midpoint of $\overline{FG}$, and Y is the midpoint of $\overline{FH}$. Show that $\overline{XY} \parallel \overline{GH}$ and $XY = \frac{1}{2}GH$.

   The coordinates of X and Y are $\left(-3, \frac{5}{2}\right)$ and $\left(-3, -\frac{1}{2}\right)$. The slope of $\overline{XY}$ and $\overline{GH}$ is undefined, so the lines are parallel. The length of $\overline{XY}$ is 3 and the length of $\overline{GH}$ is 6, so $XY = \frac{1}{2}GH$.

4. One of the vertices of $\triangle PQR$ is $P(3, -2)$. The midpoint of $\overline{PQ}$ is $M(4, 0)$. The midpoint of $\overline{QR}$ is $N(7, 1)$. Show that $\overline{MN} \parallel \overline{PR}$ and $MN = \frac{1}{2}PR$.

   $\frac{3+x}{2} = 4$, So $x = 5$; $\quad \frac{-2+y}{2} = 0$, So $y = 2$.

   The coordinates of Q are $(5, 2)$.

   $\frac{5+x}{2} = 7$, So $x = 9$; $\quad \frac{2+y}{2} = 1$, So $y = 0$.

   The coordinates of R are $(9, 0)$.

   The slope of $\overline{MN}$ and $\overline{PR}$ is $\frac{1}{3}$, so the lines are parallel. The length of $\overline{MN}$ is $\sqrt{10}$ and the length of $\overline{PR}$ is $2\sqrt{10}$, so $MN = \frac{1}{2}PR$.

LANGUAGE SUPPORT EL

Communicate Math

Divide students into pairs. Provide each pair with pictures of triangles with segments joining two sides. Have one student explain why a drawing does or does not show a midsegment. Encourage the use of the words *parallel* and *midpoint*. Have students measure the distance between the segment and the base at different intervals to see if the segment and the base are parallel, and to measure the sides joined by the segment, to see if the segment's endpoints are the midpoints of the sides. Students switch roles and repeat the process.

5. One of the vertices of $\triangle ABC$ is $A(0, 0)$. The midpoint of $\overline{AC}$ is $J\left(\frac{3}{2}, 2\right)$. The midpoint of $\overline{BC}$ is $K(4, 2)$. Show that $\overline{JK} \parallel \overline{BA}$ and $JK = \frac{1}{2}BA$.

$\frac{0 + x}{2} = \frac{3}{2}$ $\quad$ $\frac{0 + y}{2} = 2$ $\quad$ So, the coordinates of C are $(3, 4)$.

$x = 3$ $\quad$ $y = 4$

Use the midpoint formula to find the coordinates of $B(x, y)$.

$\frac{3 + x}{2} = 4$ $\quad$ $\frac{4 + y}{2} = 2$

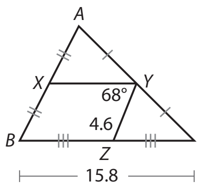
$3 + x = 8$ $\quad$ $4 + y = 4$

$x = 5$ $\quad$ $y = 0$

So, the coordinates of B are $(5, 0)$.

The slope of $\overline{JK}$ and $\overline{BA}$ is 0, so the lines are parallel. The length of $\overline{JK}$ is $\frac{5}{2}$ and the length of $\overline{BA}$ is 5, so $JK = \frac{1}{2}BA$.

Find each measure.



6. XY

$XY = \frac{1}{2}BC = \frac{1}{2}(15.8) = 7.9$

7. BZ

$BZ = \frac{1}{2}BC = \frac{1}{2}(15.8) = 7.9$

8. AX

Since $AX = \frac{1}{2}AB$, find AB first.

$AB = 2(YZ) = 2(4.6) = 9.2$

$AX = \frac{1}{2}AB = \frac{1}{2}(9.2) = 4.6$

9. $m\angle YZC$

$\angle YZC$ and $\angle XYZ$ are alternate interior angles, so they are congruent.

$m\angle YZC = 68°$

10. $m\angle BXY$

$m\angle BXY + m\angle ZYX = 180°$ (same-side interior angles)

$m\angle BXY + 68° = 180°$

$m\angle BXY = 112°$

EVALUATE

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Investigating Midsegments of a Triangle	Exercises 1–3
Example 1 Describing Midsegments on a Coordinate Grid	Exercises 4–5
Example 2 Using the Triangle Midsegment Theorem	Exercises 6–15

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Remind students of the key concepts they need to do the exercises. Define *midsegment*. Make a sketch on the board showing students how to draw a midsegment, and point out that every triangle has three midsegments. Add that, in this lesson, students will primarily work with one midsegment at a time.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1–15	2 Skills/Concepts	MP.6 Precision
16–21	2 Skills/Concepts	MP.2 Reasoning
22	3 Strategic Thinking H.O.T.	MP.4 Modeling
23	3 Strategic Thinking H.O.T.	MP.6 Precision
24	3 Strategic Thinking H.O.T.	MP.4 Modeling

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Discuss each of the following statements as a class. Have students explain how the Triangle Midsegment Theorem justifies their responses.

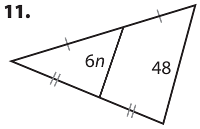
1. A triangle has three midsegments.

2. The midsegments form a triangle with a perimeter equal to one-half the perimeter of the original triangle.

3. The midsegments form a triangle with area equal to one-fourth the area of the original triangle.

Algebra **Find the value of n in each triangle.**

11.

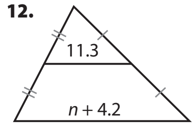


$6n = \frac{1}{2}(48)$

$6n = 24$

$n = 4$

12.

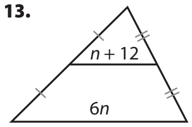


$2(11.3) = n + 4.2$

$22.6 = n + 4.2$

$18.4 = n$

13.



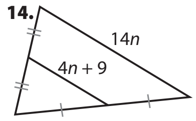
$6n = 2(n + 12)$

$6n = 2n + 24$

$4n = 24$

$n = 6$

14.



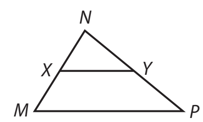
$14n = 2(4n + 9)$

$14n = 8n + 18$

$6n = 18$

$n = 3$

15. Line segment XY is a midsegment of $\triangle MNP$. Determine whether each of the following statements is true or false. Select the correct answer for each lettered part.



a. $MP = 2XY$	● True	○ False
b. $MP = \frac{1}{2}XY$	○ True	● False
c. $MX = XN$	● True	○ False
d. $MX = \frac{1}{2}NX$	○ True	● False
e. $NX = YN$	○ True	● False
f. $XY = \frac{1}{2}MP$	● True	○ False

16. What do you know about two of the midsegments in an isosceles triangle? Explain.

Two of the midsegments will be congruent because their lengths are $\frac{1}{2}$ the length of congruent sides.

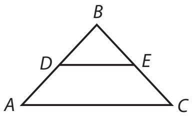
17. Suppose you know that the midsegments of a triangle are all 2 units long. What kind of triangle is it?

an equilateral triangle with sides 4 units long

18. In $\triangle ABC$, $m\angle A = 80°$, $m\angle B = 60°$, $m\angle C = 40°$. The midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ are D, E, and F, respectively. Which midsegment will be the longest? Explain how you know.

$\overline{BC}$ is the longest side because it is opposite the largest angle, so midsegment $\overline{DF}$ will be the longest.

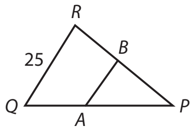
19. **Draw Conclusions** Carl's Construction is building a pavilion with an A-frame roof at the local park. Carl has constructed two triangular frames for the front and back of the roof, similar to $\triangle ABC$ in the diagram. The base of each frame, represented by $\overline{AC}$, is 36 feet long. He needs to insert a crossbar connecting the midpoints of $\overline{AB}$ and $\overline{BC}$, for each frame. He has 32 feet of timber left after constructing the front and back triangles. Is this enough to construct the crossbar for both the front and back frame? Explain.



No. If $\overline{AC}$ is 36 feet long, $\overline{DE}$ is 18 feet long. Because he needs to insert two crossbars, he needs 2×18, or 36, feet of timber. He needs 4 more feet.

20. **Critique Reasoning** Line segment AB is a midsegment in $\triangle PQR$. Kayla calculated the length of $\overline{AB}$. Her work is shown below. Is her answer correct? If not, explain her error.

$$2(QR) = AB$$
$$2(25) = AB$$
$$50 = AB$$



No, it is not correct. The equation should have $\frac{1}{2}$ QR on the left instead of $2(QR)$.

21. Using words or diagrams, tell how to construct a midsegment using only a straightedge and a compass.

Possible answer: First construct a triangle with a straightedge. Next use the compass to find the midpoint of two sides of the triangle. Finally, connect the two midpoints to create a midsegment parallel to the third side.

PEER-TO-PEER DISCUSSION

Ask students to discuss with a partner how to locate the midsegments of a triangle drawn in the coordinate plane. Have them use the slope formula or distance formula and the Triangle Midsegment Theorem to verify that the midsegments are equal to one-half the third side and are parallel to the third side. Have them draw examples to justify their reasoning.

AVOID COMMON ERRORS

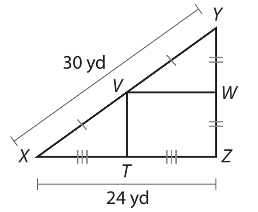
When using triangles in the coordinate plane, some students may assume that the Triangle Midsegment Theorem applies only to acute, scalene triangles. Have students work in groups to draw other types of triangles in the coordinate plane and use algebraic expressions to write the triangle relationships that apply to these triangles.

JOURNAL

Have students explain how to find the length of a midsegment when given the length of the parallel side.

H.O.T. Focus on Higher Order Thinking

22. Multi–Step A city park will be shaped like a right triangle, and there will be two pathways for pedestrians, shown by $\overline{VT}$ and $\overline{VW}$ in the diagram. The park planner only wrote two lengths on his sketch as shown. Based on the diagram, what will be the lengths of the two pathways?



$VW = \frac{1}{2}(XZ) = \frac{1}{2}(24) = 12$

$24^2 + (YZ)^2 = 30^2$ (Pyth. Thm.)

$576 + (YZ)^2 = 900$

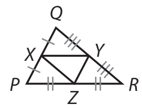
$(YZ)^2 = 324$

$YZ = 18$

$VT = \frac{1}{2}(YZ) = \frac{1}{2}(18) = 9$

$VW = 12$ yd; $VT = 9$ yd

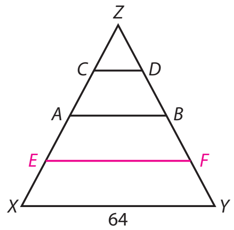
23. Communicate Mathematical Ideas $\triangle XYZ$ is the midsegment of $\triangle PQR$. Write a congruence statement involving all four of the smaller triangles. What is the relationship between the area of $\triangle XYZ$ and $\triangle PQR$?



$\triangle QXY \cong \triangle XPZ \cong \triangle YZR \cong \triangle ZYX$;

area of $\triangle XYZ = \frac{1}{4}$ area of $\triangle PQR$

24. Copy the diagram shown. $\overline{AB}$ is a midsegment of $\triangle XYZ$. $\overline{CD}$ is a midsegment of $\triangle ABZ$.



a. What is the length of $\overline{AB}$? What is the ratio of AB to XY?

$AB = \frac{1}{2}(XY)$, so AB is 32. The ratio of AB to XY is 32:64, or 1:2.

b. What is the length of $\overline{CD}$? What is the ratio of CD to XY?

$CD = \frac{1}{2}(AB)$, so CD is 16. The ratio of CD to XY is 16:64, or 1:4.

c. Draw $\overline{EF}$ such that points E and F are $\frac{3}{4}$ the distance from point Z to points X and Y. What is the ratio of EF to XY? What is the length of $\overline{EF}$?

$EF = \frac{3}{4}(XY) = \frac{3}{4}(64) = 48$

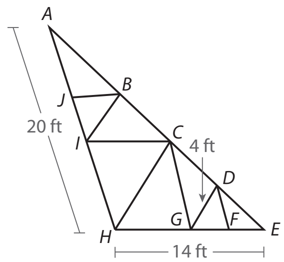
The length of $\overline{EF}$ is 48, so the ratio of EF to XY is 48:64, or 3:4.

d. Make a conjecture about the length of non-midsegments when compared to the length of the third side.

The length of the non-midsegment compared to the length of the third side is the same as the ratio of the distance of the segment from the vertex opposite the third side compared to the whole triangle.

Lesson Performance Task

The figure shows part of a common roof design using very strong and stable triangular *trusses*. Points B, C, D, F, G, and I are midpoints of $\overline{AC}$, $\overline{AE}$, $\overline{CE}$, $\overline{GE}$, $\overline{HE}$ and $\overline{AH}$ respectively. What is the total length of all the stabilizing bars inside $\triangle AEH$? Explain how you found the answer.



By the Midsegment Theorem, $IC = 7$ ft and $CG = 10$ ft because they are, respectively, half of HE and AH. $CH = 8$ ft because it is twice DG. $DF = 5$ ft, $JB = 3.5$ ft, and $IB = 4$ ft because they are, respectively, half of CG, IC, and HC.

Total length: $JB + IB + IC + CH + CG + DG + DF = 3.5 + 4 + 7 + 8 + 10 + 4 + 5 = 41.5$ ft

EXTENSION ACTIVITY

Have students research tri-bearing roof trusses. Among topics they can report on:

- the definition of tri-bearing roof truss;
- the design of such a truss, together with a drawing and typical measurements;
- the meaning of the term *tri-bearing;*
- a description of the relationship between tri-bearing roof trusses and the Midsegment Theorem.

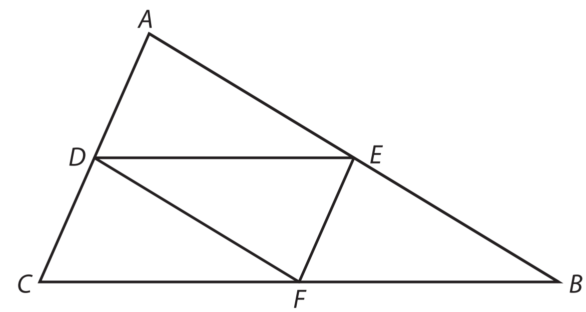
AVOID COMMON ERRORS

Students may mistakenly assume that because B, C, and D divide $\overline{AE}$ into four congruent segments, G and F must divide $\overline{HE}$ into three congruent segments, and that J and I divide $\overline{AH}$ into three congruent segments. Point out that because G is the midpoint of $\overline{HE}$, GF and FE are each only half of HG. Similarly, because I is the midpoint of $\overline{AH}$, AJ and JI are each only half of IH.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 On the roof truss diagrammed below, D, E, and F are the midpoints of, respectively, $\overline{AC}$, $\overline{AB}$, and $\overline{CB}$.



Compare the perimeter and area of $\triangle ABC$ with those of $\triangle DEF$. Explain how you found your answers.

Students can use the Midsegment Theorem and the facts that $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$ are parallel to and half the length of, respectively, $\overline{CB}$, $\overline{AC}$, and $\overline{AB}$ to establish that (1) the perimeter of $\triangle DEF$ is half that of $\triangle ABC$, and (2) the four small triangles in the diagram are congruent, which means that the area of $\triangle DEF$ is one-fourth that of $\triangle ABC$.

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain.

0 points: Student does not demonstrate understanding of the problem.